## Diffusion Models for Inverse Problems Done Right

## Ju Sun

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Conference on Computational Science and Engineering

Generative Machine Learning Models for Uncertainty Quantification



Inverse Problems

#### Inverse problems

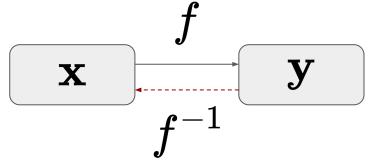
#### Inverse problem: given $\mathbf{y}\,=\,f(\mathbf{x})$ , recover $\mathbf{x}$

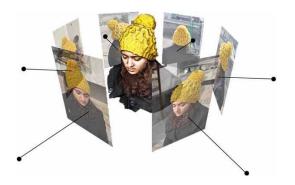


Image denoising

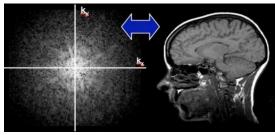


Image super-resolution

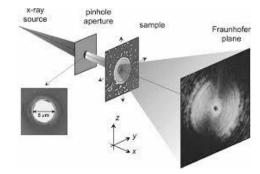




3D reconstruction



**MRI** reconstruction



Coherent diffraction imaging (CDI)

### Traditional methods

Inverse problem: given  $\mathbf{y} = f(\mathbf{x})$ , recover  $\mathbf{x}$ 

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}} \quad \mathsf{RegFit}$$

Questions

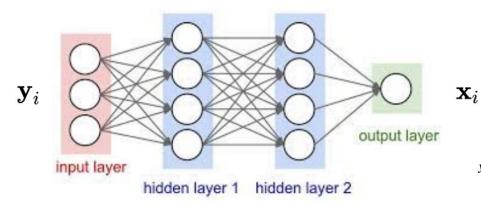
- Which  $\ell$ ? (e.g., unknown/compound noise)
- Which R? (e.g., structures not amenable to math description)
- Speed

## Deep learning has changed everything

## With paired datasets $\{(\boldsymbol{y}_i, \boldsymbol{x}_i)\}_{i=1,...,N}$

#### **Direct inversion**

Learn 
$$f^{-1}$$
from  $\{(oldsymbol{y}_i,oldsymbol{x}_i)\}_{i=1,...,N}$ 

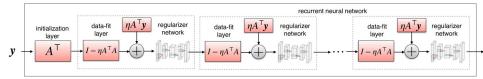


#### Algorithm unrolling

$$\min_{\mathbf{x}} \ell(\mathbf{y}, f(\mathbf{x})) + \lambda \ \mathrm{R}(\mathbf{x})$$

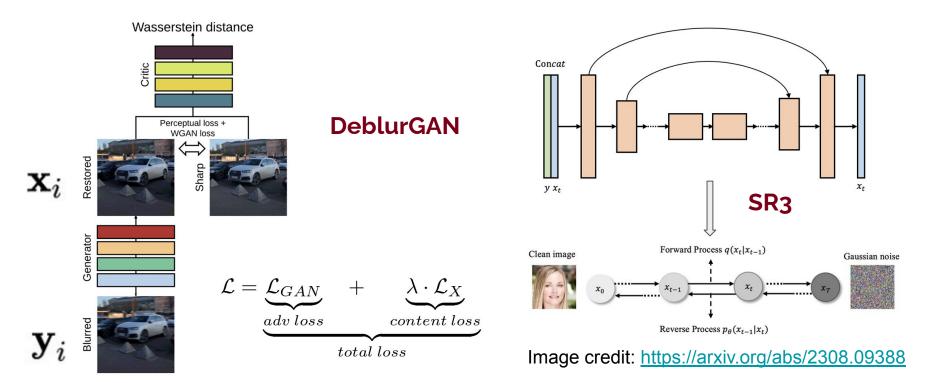
$$\mathbf{x}^{k+1} \,=\, \mathcal{P}_Rig(\mathbf{x}^k\,-\,\eta
abla^ op fig(\mathbf{x}^kig)\ell'ig(\mathbf{y},\,f(\mathbf{x}^k)ig)ig)$$

Idea: make  $\mathcal{P}_R$  trainable



With paired datasets  $\{(\boldsymbol{y}_i, \boldsymbol{x}_i)\}_{i=1,...,N}$ 

#### **Conditional generation & regularization**

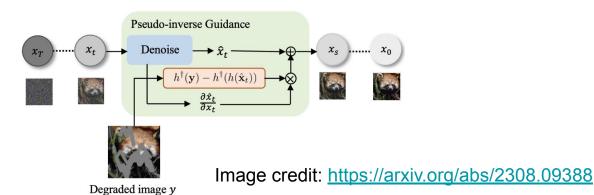


## With object datasets only $\{m{x}_i\}_{i=1,...,N}$

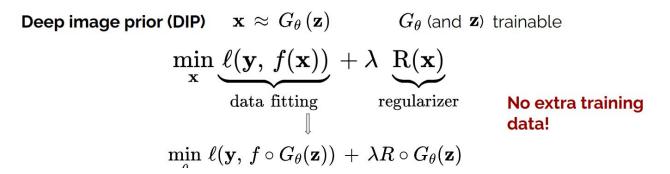
# Model the distribution of the objects first, and then plug the prior in GAN Inversion

 $\begin{array}{ll} { { \mathsf{Pretraining:}} \ \, { { \mathbf{x} } }_i \ \approx \ G_\theta \left( { { \mathbf{z} } _i } \right) \ \forall \, i } \\ { { \mathsf{Deployment:}} \ \, \min_{ { \mathbf{z} } } \ \, \ell ( { \mathbf{y} }, \, f \circ G_\theta ( { \mathbf{z} } ) ) \ + \ \lambda R \ \circ G_\theta \left( { \mathbf{z} } \right) } \end{array} \end{array}$ 

Interleaving pretrained diffusion models



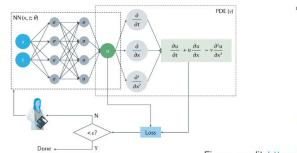
#### Without datasets? Single-instance methods



Neural implicit representation (NIR)

 $\mathbf{x} \approx \mathcal{D} \circ \overline{\mathbf{x}}$   $\mathcal{D}$ : discretization  $\overline{\mathbf{x}}$ : continuous function

Physics-informed neural networks (PINN)



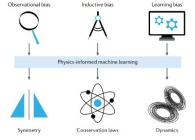


Figure credit: https://www.nature.com/articles/s42254-021-00314-5

Table 2: Major categories of methods learning to solve inverse problems based on what is known about the forward model A and the nature of the training data, with examples for each. Details are described throughout Section 4.

anougnour beetion 4.	Supervised with matched $(x,y)$ pairs	Train from un- paired x's and y's (Unpaired ground truths and Measure- ments)	Train from <i>x</i> 's only (Ground truth only)	Train from y's only (Measure- ments only)
A fully known during training and testing (§4.1)	§4.1.1: Denoising auto-encoders [16], U-Net [78], Deep convolutional framelets [79] Unrolled opti- mization [80–83], Neumann net- works [84]	amounts to training from $(x, y)$ pairs	amounts to training from $(x, y)$ pairs	§4.1.2: SURE LDAMP [85, 86], Deep Basis Pur- suit [87]
A known only at test time (§4.2)	§4.2.2	§4.2.2	§4.2.1: CSGM [25], LDAMP [88], OneNet [22], Plug- and-play [89], RED [90]	§4.2.2
A partially known (§4.3)	§4.3.1	§4.3.2: CycleGAN [91]	§4.3.3: Blind de- convolution with GAN's [92–94]	§4.3.4: Ambi- entGAN [76], Noise2Noise [95], UAIR [96]
$\mathcal{A}$ unknown (§4.4)	§4.4.1: AUTOMAP [97]	§4.4.2	§4.4.2	§4.4.2

#### Deep Learning Techniques for Inverse Problems in Imaging

Gregory Ongie<sup>\*</sup>, Ajil Jalal<sup>†</sup>, Christopher A. Metzler<sup>‡</sup> Richard G. Baraniuk<sup>§</sup>, Alexandros G. Dimakis,<sup>¶</sup> Rebecca Willett<sup>||</sup>

#### https://arxiv.org/abs/2005.06001 But focused on linear IPs

### Other specialized surveys

#### Algorithm Unrolling: Interpretable, Efficient Deep Learning for Signal and Image Processing

Vishal Monga, Senior Member, IEEE, Yuelong Li, Member, IEEE, and Yonina C. Eldar, Fellow, IEEE

#### Focused on alg. unrolling

Untrained Neural Network Priors for Inverse

Ismail Alkhouri, Evan Bell, Avrajit Ghosh, Shijun Liang, Rongrong Wang,

#### Theoretical Perspectives on Deep Learning Methods in Inverse Problems **Focused on theories for linear IPs**

Jonathan Scarlett, Reinhard Heckel, Miguel R. D. Rodrigues, Paul Hand, and Yonina C. Eldar

## This talk:

## Solving Inverse Problems (IPs) Using Pretrained Diffusion Models

#### Diffusion models

Data

$$d\boldsymbol{x} = -\beta_t/2 \cdot \boldsymbol{x}dt + \sqrt{\beta_t}d\boldsymbol{w},$$

Fixed forward diffusion process



Generative reverse denoising process

$$d\boldsymbol{x} = -\beta_t \left[ \boldsymbol{x}/2 + \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) \right] dt + \sqrt{\beta_t} d\overline{\boldsymbol{w}}.$$
  
$$\cong \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{(t)}(\boldsymbol{x})$$

#### Diffusion models for inverse problems (IPs)

Supervised

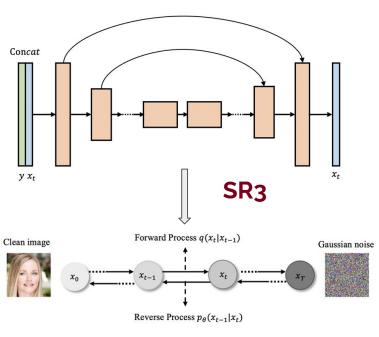
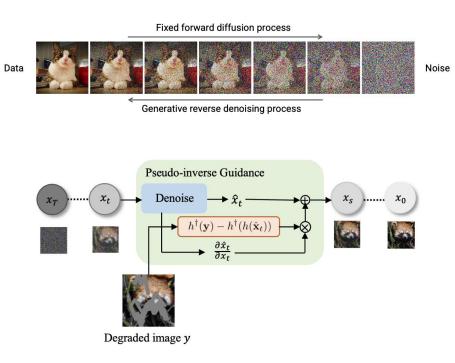


Image credit: <u>https://arxiv.org/abs/2308.09388</u>

**Zero-shot** 



#### Focus: IPs with pretrained diffusion models

(Reverse SDE for DDPM)  $d\boldsymbol{x} = -\beta_t \left[ \boldsymbol{x}/2 + \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) \right] dt + \sqrt{\beta_t} d\overline{\boldsymbol{w}}$  $\int \mathbf{T} hink of \mathbf{conditional score function}$  $\nabla_{\mathbf{x}} \log p_t(\mathbf{x} | \mathbf{y}) = \nabla_{\mathbf{x}} \log p_t(\mathbf{x}) + \nabla_{\mathbf{x}} \log p_t(\mathbf{y} | \mathbf{x})$ Conditional reverse SDE  $d\boldsymbol{x} = \left[-\beta_t/2 \cdot \boldsymbol{x} - \beta_t (\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) + \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{y}|\boldsymbol{x}))\right] dt + \sqrt{\beta_t} d\overline{\boldsymbol{w}}$ 

#### Coping with conditional score function

$$\nabla_{\boldsymbol{x}} \log p_{t}(\boldsymbol{x}|\boldsymbol{y}) = \nabla_{\boldsymbol{x}} \log p_{t}(\boldsymbol{x}) + \nabla_{\boldsymbol{x}} \log p_{t}(\boldsymbol{y}|\boldsymbol{x})$$

$$\cong \varepsilon_{\boldsymbol{\theta}}^{(t)}(\boldsymbol{x})$$

$$p_{t}(\boldsymbol{y}|\boldsymbol{x}(t))$$

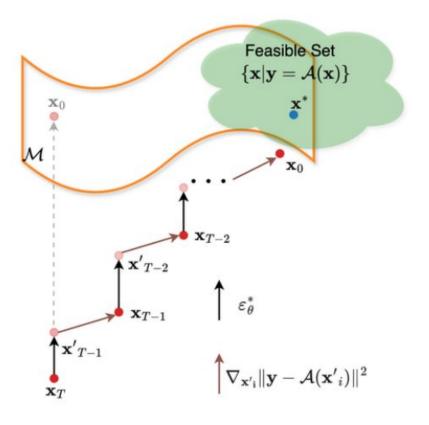
$$\stackrel{\mathbf{y}}{=} p_{t}(\boldsymbol{y}|\boldsymbol{\hat{x}}(0)[\boldsymbol{x}(t)])$$

$$\stackrel{\mathbf{y}}{=} p_{t}(\boldsymbol{y}|\boldsymbol{\hat{x}}(0)[\boldsymbol{x}(t)])$$

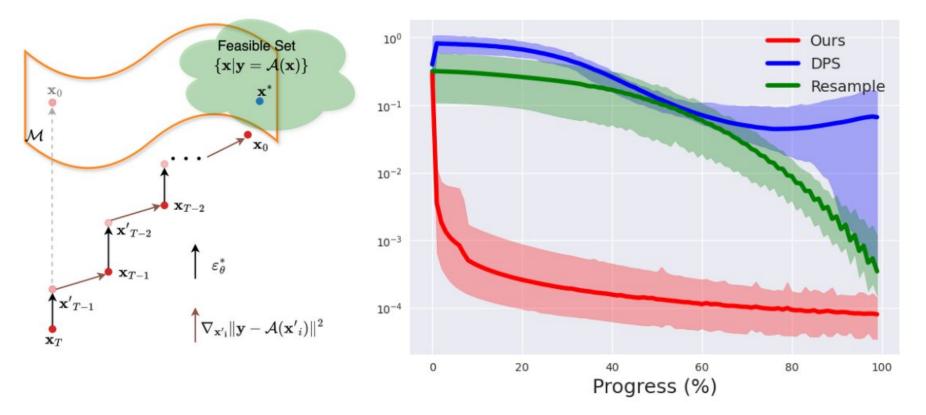
ack intractable, blue dotted line: sonu mie. tractable in general.

### Interleaving methods

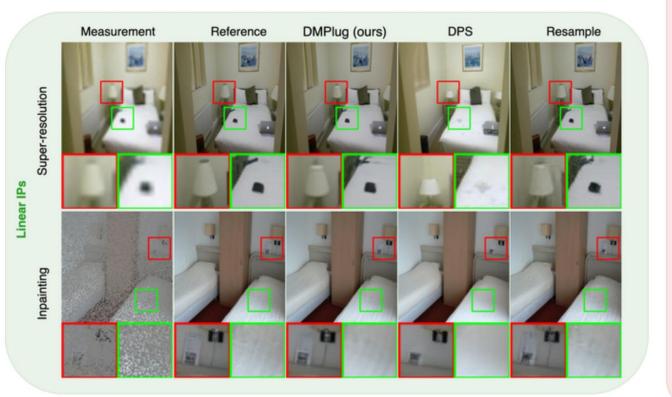
**Algorithm 1** Template for interleaving methods **Input:** # Diffusion steps T, measurement y1:  $\boldsymbol{x}_T \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ 2: for i = T - 1 to 0 do 3:  $\hat{s} \leftarrow \boldsymbol{\varepsilon}_{\boldsymbol{\rho}}^{(i)}(\boldsymbol{x}_i)$ 4:  $\hat{\boldsymbol{x}}_0 \leftarrow \frac{1}{\sqrt{\bar{\alpha}_i}} \left( \boldsymbol{x}_i - \sqrt{1 - \bar{\alpha}_i} \hat{\boldsymbol{s}} \right)$ 5:  $x'_{i-1} \leftarrow \text{DDIM}$  reverse with  $\hat{x}_0$  and  $\hat{s}$  $x_{i-1} \leftarrow (\text{Approximately})$  Projec-6: tion 39 30 33 32 40 41 34 or gradient update [20, 28, 19, 21, 29, 27, 26] with  $\hat{x}_0$ and  $x'_{i-1}$  to get closer to  $\{x|y = \mathcal{A}(x)\}$ 7: end for **Output:** Recovered object  $x_0$ 



#### Issue I: Measurement feasibility



## Issue 2: Manifold feasibility



# BID **BID** with turbulence Measurement Reference DMPlug (ours) BlindDPS Stripformer

**Nonlinear IPs** 

#### Issue 3: Robustness to unknown noise

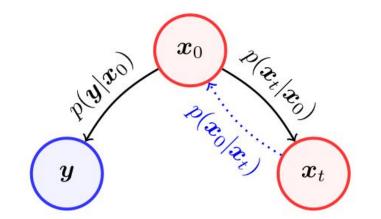
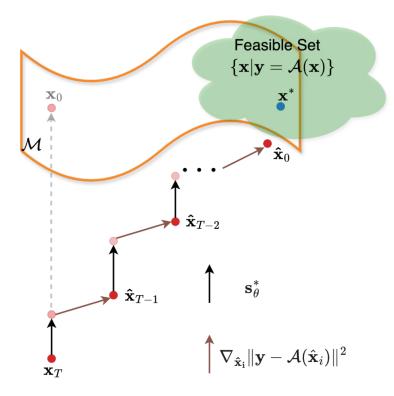


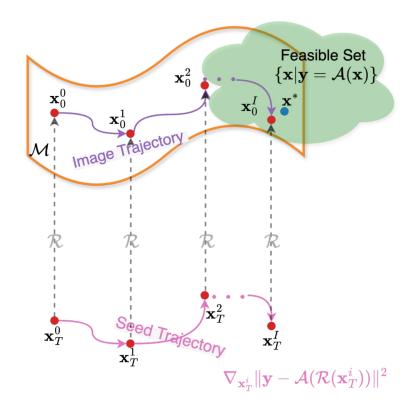
Figure 2: Probabilistic graph. Black solid line: tractable, blue dotted line: in-tractable in general.

Algorithm 1 DPS - Gaussian **Require:** N, y,  $\{\zeta_i\}_{i=1}^N, \{\tilde{\sigma}_i\}_{i=1}^N$ 1:  $\boldsymbol{x}_N \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ 2: for i = N - 1 to 0 do 3:  $\hat{\boldsymbol{s}} \leftarrow \boldsymbol{s}_{\theta}(\boldsymbol{x}_{i}, i)$ 4:  $\hat{\boldsymbol{x}}_{0} \leftarrow \frac{1}{\sqrt{\bar{\alpha}_{i}}}(\boldsymbol{x}_{i} + (1 - \bar{\alpha}_{i})\hat{\boldsymbol{s}})$ 5:  $\boldsymbol{z} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ 6:  $x'_{i-1} \leftarrow \frac{\sqrt{\alpha_i}(1-\bar{\alpha}_{i-1})}{1-\bar{\alpha}_i} x_i + \frac{\sqrt{\bar{\alpha}_{i-1}\beta_i}}{1-\bar{\alpha}_i} \hat{x}_0 + \tilde{\sigma}_i z$ 7:  $\boldsymbol{x}_{i-1} \leftarrow \boldsymbol{x}_{i-1}' - \zeta_i \nabla_{\boldsymbol{x}_i} \| \boldsymbol{y} - \mathcal{A}(\hat{\boldsymbol{x}}_0) \|_2^2$ 8: end for 9: return  $\hat{\mathbf{x}}_0$ 

depending on noise level

#### Our solution: DMPlug





#### Our solution: DMPlug

#### Viewing the reverse process as a function ${\cal R}$

$$\mathcal{R} = g_{\varepsilon_{\theta}^{(0)}} \circ g_{\varepsilon_{\theta}^{(1)}} \circ \cdots \circ g_{\varepsilon_{\theta}^{(T-2)}} \circ g_{\varepsilon_{\theta}^{(T-1)}}. \quad (\circ \text{ means function composition})$$

$$(\mathbf{DMPlug}) \ \boldsymbol{z}^{*} \in \arg\min_{\boldsymbol{z}} \ \ell(\boldsymbol{y}, \mathcal{A}(\mathcal{R}(\boldsymbol{z}))) + \Omega(\mathcal{R}(\boldsymbol{z})), \qquad \boldsymbol{x}^{*} = \mathcal{R}(\boldsymbol{z}^{*}).$$

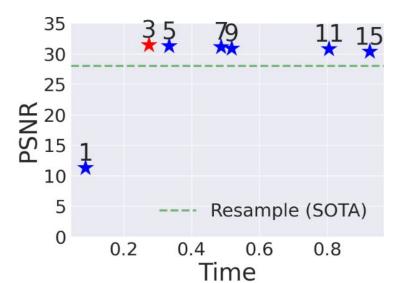
$$(\mathbf{Manifold}_{feasibility})$$

$$(\mathbf{Manifold}_{feasibility})$$

#### Overcoming the computational bottleneck

$$\begin{split} \mathcal{R} &= g_{\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{(0)}} \circ g_{\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{(1)}} \circ \cdots \circ g_{\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{(T-2)}} \circ g_{\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{(T-1)}}. \quad (\circ \text{ means function composition}) \\ (\mathbf{DMPlug}) \ \boldsymbol{z}^* \in \arg\min_{\boldsymbol{z}} \ \ell(\boldsymbol{y}, \mathcal{A}(\mathcal{R}(\boldsymbol{z}))) + \Omega(\mathcal{R}(\boldsymbol{z})), \qquad \boldsymbol{x}^* = \mathcal{R}(\boldsymbol{z}^*). \end{split}$$

**Issue**: T blocks of DNNs involved, and we have to back-propagate through it



#### On linear IPs

Table 1: (Linear IPs) Super-resolution and inpainting with additive Gaussian noise ( $\sigma = 0.01$ ). (Bold: best, <u>under</u>: second best, green: performance increase, red: performance decrease)

	Super-resolution $(4 \times)$					<b>Inpainting (Random</b> 70%)						
	<b>CelebA</b> [65] (256 × 256)		<b>FFHQ 66</b> (256 × 256)			<b>CelebA 65</b> (256 × 256)			<b>FFHQ 66</b> (256 × 256)			
	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑
ADMM-PnP 68	0.217	26.99	0.808	0.229	26.25	0.794	0.091	31.94	0.923	0.104	30.64	0.901
DMPS 29	<u>0.070</u>	28.89	0.848	0.076	28.03	0.843	0.297	24.52	0.693	0.326	23.31	0.664
DDRM 32	0.226	26.34	0.754	0.282	25.11	0.731	0.185	26.10	0.712	0.201	25.44	0.722
MCG 30	0.725	19.88	0.323	0.786	18.20	0.271	1.283	10.16	0.049	1.276	10.37	0.050
ILVR 41	0.322	21.63	0.603	0.360	20.73	0.570	0.447	15.82	0.484	0.483	15.10	0.450
DPS 19	0.087	28.32	0.823	0.098	27.44	0.814	0.043	32.24	0.924	0.046	30.95	0.913
ReSample 20	0.080	28.29	0.819	0.108	25.22	0.773	0.039	30.12	0.904	0.044	27.91	0.884
DMPlug (ours)	0.067	31.25	0.878	<u>0.079</u>	30.25	0.871	0.039	34.03	0.936	0.038	33.01	0.931
Ours vs. Best compe.	-0.003	+2.36	+0.030	+0.003	+2.22	+0.028	-0.000	+1.79	+0.012	-0.006	+2.06	+0.018

### On nonlinear IPs

Table 2: (Nonlinear IP) Nonlinear deblurring with additive Gaussian noise ( $\sigma = 0.01$ ). (Bold: best, <u>under</u>: second best, green: performance increase, red: performance decrease)

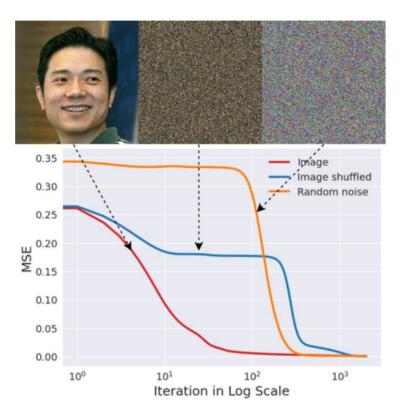
	<b>CelebA</b> [65] (256 × 256)			FFHQ	<b>66</b> (256	× 256)	<b>LSUN 67</b> (256 × 256)		
	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑
BKS-styleGAN 69	1.047	22.82	0.653	1.051	22.07	0.620	0.987	20.90	0.538
BKS-generic 69	1.051	21.04	0.591	1.056	20.76	0.583	0.994	18.55	0.481
MCG 30	0.705	13.18	0.135	0.675	13.71	0.167	0.698	14.28	0.188
ILVR 41	0.335	21.08	0.586	0.374	20.40	0.556	0.482	18.76	0.444
DPS 19	0.149	24.57	0.723	0.130	25.00	0.759	0.244	23.46	0.684
ReSample 20	<u>0.104</u>	28.52	0.839	<u>0.104</u>	27.02	0.834	0.143	26.03	0.803
<b>DMPlug</b> (ours)	0.073	31.61	0.882	0.057	32.83	0.907	0.083	30.74	0.882
Ours vs. Best compe.	-0.031	+3.09	+0.043	-0.047	+5.79	+0.073	-0.060	+4.71	+0.079

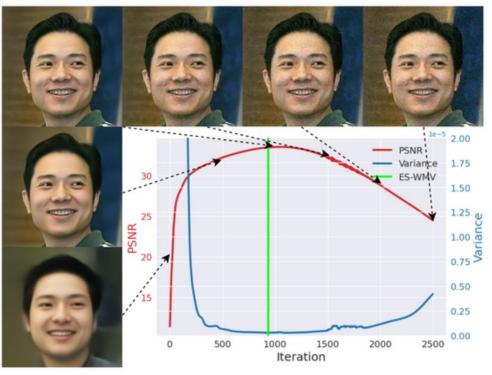
#### More on nonlinear IPs

Table 4: (Nonlinear IP) **BID** with additive Gaussian noise ( $\sigma = 0.01$ ). (**Bold**: best, <u>under</u>: second best, green: performance increase, red: performance decrease)

	<b>CelebA</b> [65] (256 × 256)					<b>FFHQ</b> 66 (256 × 256)						
	Motion blur		Gaussian blur			Motion blur			Gaussian blur			
	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑
SelfDeblur 75	0.568	16.59	0.417	0.579	16.55	0.423	0.628	16.33	0.408	0.604	16.22	0.410
DeBlurGANv2 5	0.313	20.56	0.613	0.350	24.29	0.743	0.353	19.67	0.581	0.374	23.58	0.726
Stripformer 6	0.287	22.06	0.644	0.316	25.03	<u>0.747</u>	0.324	21.31	0.613	0.339	24.34	0.728
MPRNet 7	0.332	20.53	0.620	0.375	22.72	0.698	0.373	19.70	0.590	0.394	22.33	0.685
Pan-DCP 73	0.606	15.83	0.483	0.653	20.57	0.701	0.616	15.59	0.464	0.667	20.69	0.698
Pan- $\ell_0$ 74	0.631	15.16	0.470	0.654	20.49	0.675	0.642	14.43	0.443	0.669	20.34	0.671
ILVR 41	0.398	19.23	0.520	0.338	21.20	0.588	0.445	18.33	0.484	0.375	20.45	0.555
BlindDPS 21	0.164	23.60	0.682	0.173	25.15	0.721	0.185	<u>21.77</u>	0.630	0.193	23.83	0.693
<b>DMPlug</b> (ours)	0.104	29.61	0.825	0.140	28.84	0.795	0.135	27.99	0.794	0.169	28.26	0.811
Ours vs. Best compe.	-0.060	+6.01	+0.143	-0.033	+3.69	+0.048	-0.050	+6.22	+0.164	-0.024	+3.92	+0.083

#### How to achieve robustness to unknown noise?





Early-learning-then-overfitting (OLTO)

Algorithm 3 DMPlug+ES–WMV for solving general IPs

**Input:** # diffusion steps T, y, window size W, patience P, empty queue Q, iteration counter e = 0, VAR<sub>min</sub> =  $\infty$ 1: while not stopped do for i = T - 1 to 0 do 2:  $\hat{s} \leftarrow \boldsymbol{\varepsilon}_{\boldsymbol{\rho}}^{(i)}(\boldsymbol{z}_{i}^{e})$ 3:  $\hat{m{z}}_0^e \leftarrow rac{1}{\sqrt{ar{lpha}_i}}ig(m{z}_i^e - \sqrt{1 - ar{lpha}_i} \hat{m{s}}ig)$ 4: 5:  $z_{i-1}^e \leftarrow \text{DDIM}$  reverse with  $\hat{z}_0^e, \hat{s}$ end for 6: Update  $z_T^{e+1}$  from  $z_T^e$  via a gradient update for Eq. (7) 7: push  $\mathcal{R}(\boldsymbol{z}_T^{e+1})$  to  $\mathcal{Q}$ , pop queue if  $|\mathcal{Q}| > W$ 8: 9: if  $|\mathcal{Q}| = W$  then compute VAR of elements in Q via Eq. (15) 10: 11: if VAR < VAR<sub>min</sub> then  $\text{VAR}_{\min} \leftarrow \text{VAR}, \boldsymbol{z}^* \leftarrow \boldsymbol{z}_{\tau}^{e+1}$ 12: 13: end if 14: if  $VAR_{min}$  stagnates for P iterations then 15: stop and return  $z^*$ 16: end if 17: end if 18: e = e + 119: end while **Output:** Recovered object  $\mathcal{R}(\boldsymbol{z}^*)$ 

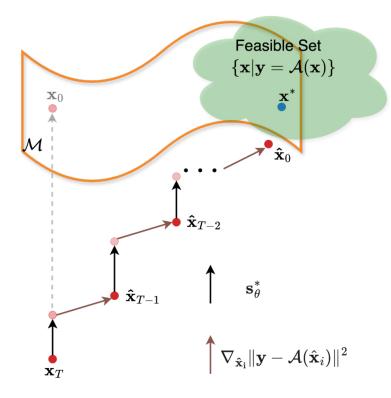
Early stopping based on running variance

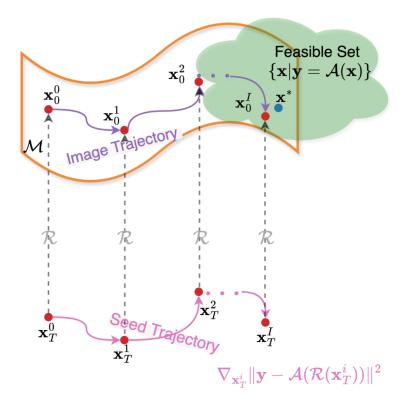
#### Robustness to unknown noise

Table 5: (**Robustness and ES**) **Super-resolution** and **nonlinear deblurring** on CelebA [65] with different types and levels of noise. We only show PSNR $\uparrow$  and PSNR Gap $\downarrow$  to save space. (**Bold**: best, <u>under</u>: second best, <u>green</u>: performance increase, red: performance decrease)

	(L	inear) Super-	resolution (4	×)	(Nonlinear) Non-uniform image deblurring				
	Gaussian	Impulse	Shot	Speckle	Gaussian	Impulse	Shot	Speckle	
	Low/High	Low/High	Low/High	Low/High	Low/High	Low/High	Low/High	Low/High	
ADMM-PnP 68	20.17/17.97	14.28/14.52	19.97/17.82	19.42/18.41	N/A	N/A	N/A	N/A	
DMPS 29	20.62/17.54	18.78/16.05	19.96/16.74	20.77/18.73	N/A	N/A	N/A	N/A	
DDRM 32	15.45/14.79	14.82/14.14	15.31/14.59	15.46/15.03	N/A	N/A	N/A	N/A	
MCG 30	17.43/15.83	16.39/15.07	17.19/15.49	17.44/16.43	12.88/12.85	13.16/13.04	13.21/13.13	13.24/13.07	
ILVR 41	21.08/21.03	20.93/20.00	21.19/21.12	20.96/20.89	21.70/21.43	21.43/21.00	21.56/21.24	21.53/21.36	
DPS 19	25.51/24.58	24.89/23.92	25.47/24.27	25.69/24.97	23.97/23.35	23.74/23.18	24.32/23.58	23.45/23.61	
ReSample 20	14.30/13.04	15.56/13.48	14.38/12.87	15.64/14.23	23.17/20.45	20.69/18.91	22.94/20.11	23.59/21.66	
BKS-styleGAN 69	N/A	N/A	N/A	N/A	22.61/22.53	22.64/22.34	22.96/22.79	22.70/22.56	
BKS-generic 69	N/A	N/A	N/A	N/A	16.85/15.09	14.86/13.44	16.69/14.74	17.04/15.99	
DMPlug (ours)	26.49/25.29	26.01/24.76	26.34/26.34	26.81/25.81	27.58/26.60	27.22/26.13	27.71/26.55	27.68/26.96	
Ours vs. Best compe.	0.98/0.71	1.12/0.84	0.87/2.07	1.12/0.84	3.61/3.25	3.48/2.95	3.39/2.97	4.23/3.35	
PSNR Gap $\downarrow$	0.36/0.46	0.38/0.60	0.25/0.49	0.20/0.21	0.15/0.12	0.14/0.13	0.10/0.19	0.12/0.09	

### DMPlug to get everything right





### The paper (NeurIPS'24)

[Submitted on 27 May 2024]

## DMPlug: A Plug-in Method for Solving Inverse Problems with Diffusion Models

#### Hengkang Wang, Xu Zhang, Taihui Li, Yuxiang Wan, Tiancong Chen, Ju Sun

Pretrained diffusion models (DMs) have recently been popularly used in solving inverse problems (IPs). The existing methods mostly interleave iterative steps in the reverse diffusion process and iterative steps to bring the iterates closer to satisfying the measurement constraint. However, such interleaving methods struggle to produce final results that look like natural objects of interest (i.e., manifold feasibility) and fit the measurement (i.e., measurement feasibility), especially for nonlinear IPs. Moreover, their capabilities to deal with noisy IPs with unknown types and levels of measurement noise are unknown. In this paper, we advocate viewing the reverse process in DMs as a function and propose a novel plug-in method for solving IPs using pretrained DMs, dubbed DMPlug. DMPlug addresses the issues of manifold feasibility and measurement feasibility in a principled manner, and also shows great potential for being robust to unknown types and levels of noise. Through extensive experiments across various IP tasks, including two linear and three nonlinear IPs, we demonstrate that DMPlug consistently outperforms state-of-the-art methods, often by large margins especially for nonlinear IPs. The code is available at this https URL. https://arxiv.org/abs/2405.16749

## DMPlug for video restoration

Super-resolution



Temporal degradation

Inpainting (random)



Temporal degradation + Motion deblur



Table 7. Ablation study on two essential components for multilevel temporal consistency, performed on DAVIS dataset for video super-resolution  $\times 4$ . (**Bold**: best, <u>under</u>: second best)

Method	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	$WE(10^{-2})\downarrow$
SOTA [9]	26.037	0.717	0.339	1.411
Base	24.701	0.612	0.366	1.398
Base + Semantic	26.098	0.703	0.410	1.057
Base + Pixel	27.141	0.736	0.301	0.943
Base + Semantic + Pixel	27.959	0.790	0.321	0.725

Wang et al. Temporal-Consistent Video Restoration with Pre-trained Diffusion Models. Forthcoming, 2025

#### Train diffusion models in small-data regime?



(a) Full Gaussian (ambient dimension d = 12288)



(a) Full Gaussian (ambient dimension d = 16384, FID-50K=5.09)



(b) Restricted Gaussian (subspace dimension k = 2048)



(b) Restricted Gaussian (subspace dimension k = 8192, FID-50K=3.21)

Luo et al. Small-Data Flow Matching. Forthcoming, 2025