

# Robust Deep Learning: Where Are We?

Ju Sun (Computer Sci. & Eng.)

PSU-Purdue-UMD Joint Seminar on Mathematical Data Science  
Nov 6th, 2023



UNIVERSITY OF MINNESOTA

Driven to Discover<sup>SM</sup>

# Success of deep learning (DL) not news anymore

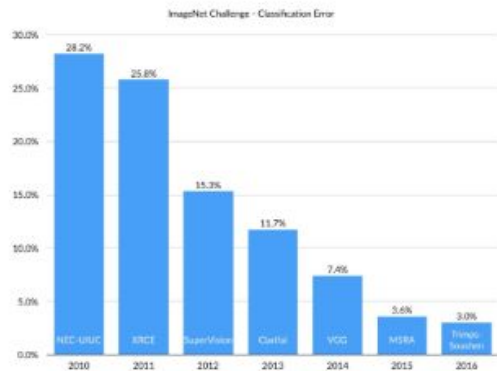


image classification

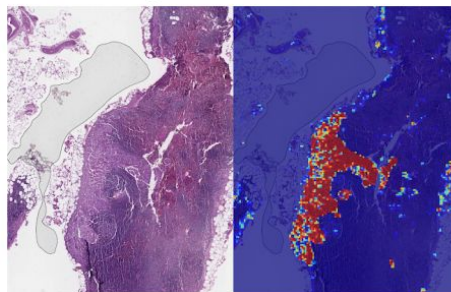


Go game (2017)

## Commercial breakthroughs ...



self-driving vehicles credit: wired.com



healthcare credit: Google AI



smart-home devices credit: Amazon



robotics credit: Cornell U.

# Robustness issues of DL not news anymore



"panda"

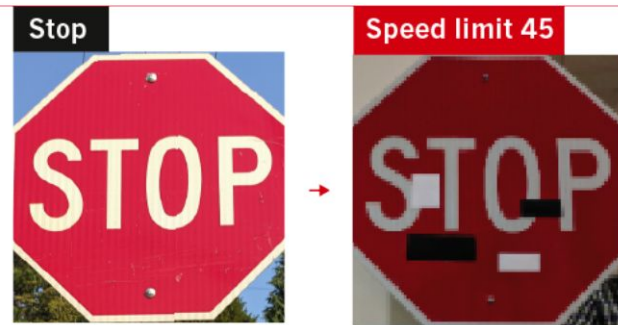
$x$

$\delta$

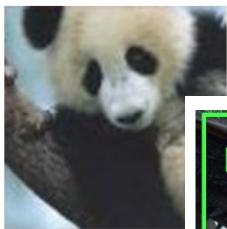
## FOOLING THE AI

Deep neural networks (DNNs) are brilliant at image recognition — but they can be easily hacked.

These stickers made an artificial-intelligence system read this stop sign as 'speed limit 45'.

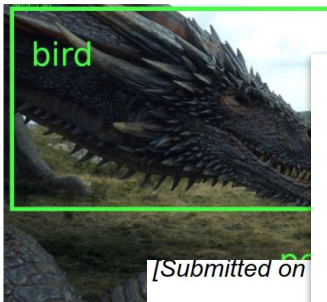


# Robustness issues across domains/tasks



"panda"

$x$



[Submitted on 06/01/2018]

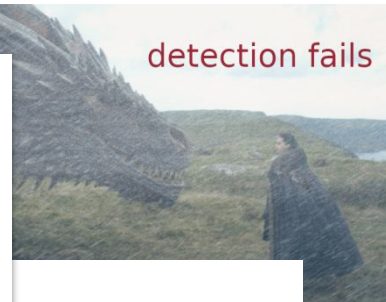
## A Multilingual Inputs

Akshay Srinivasan

Adversarial  
multilingual  
input. Our

man and Hindi. While exact results differ depending on language/datasets, our key findings from these experiments can be summarized as follows:

1. NER models for all three languages are sensitive to adversarial input.
2. Adversarial fine-tuning and re-training could improve the performance of NER models both on original and adversarial test sets, without requiring additional manual labeled data.



## Adversarial

we performed a  
small perturbations in the  
(German and Hindi) are

## Name entry Recognition

are not very robust to such changes, as indicated by the fluctuations in the overall F1 score as well as in a more fine-grained evaluation. With that knowledge, we further explored whether it is possible to improve the existing NER

# Robustness issues across models

Tutorial

## Foundational Robustness of Foundation Models

NeurIPS 2022

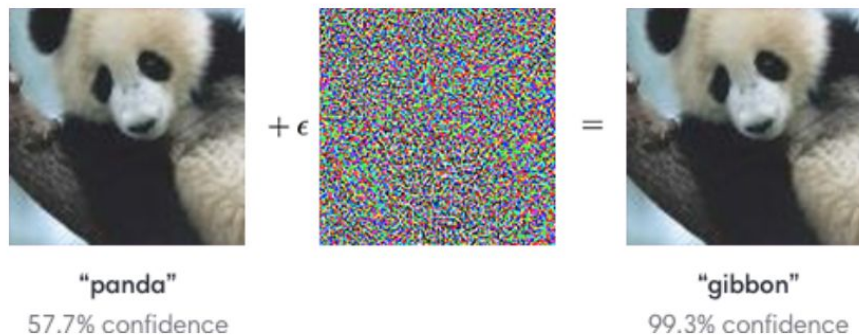
### Abstract

Foundation models adopting the methodology of deep learning with pre-training on large-scale unlabeled data and finetuning with task-specific supervision are becoming a mainstream technique in machine learning. Although foundation models hold many promises in learning general representations and few-shot/zero-shot generalization across domains and data modalities, at the same time they raise unprecedented challenges and considerable risks in robustness and privacy due to the use of the excessive volume of data and complex neural network architectures. This tutorial aims to deliver a Coursera-like online tutorial containing comprehensive lectures, a hands-on and interactive Jupyter/Colab live coding demo, and a panel discussion on different aspects of trustworthiness in foundation models. More information can be found at <https://sites.google.com/view/neurips2022-frfm-tutorial>

<https://research.ibm.com/publications/foundational-robustness-of-foundation-models>

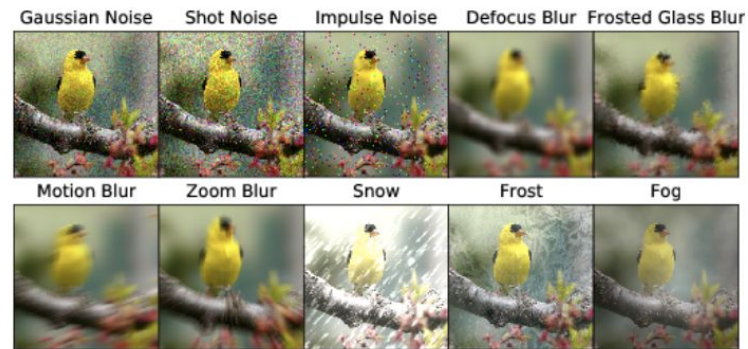


# Two kinds of robustness



credit: openai.com

## Adversarial examples

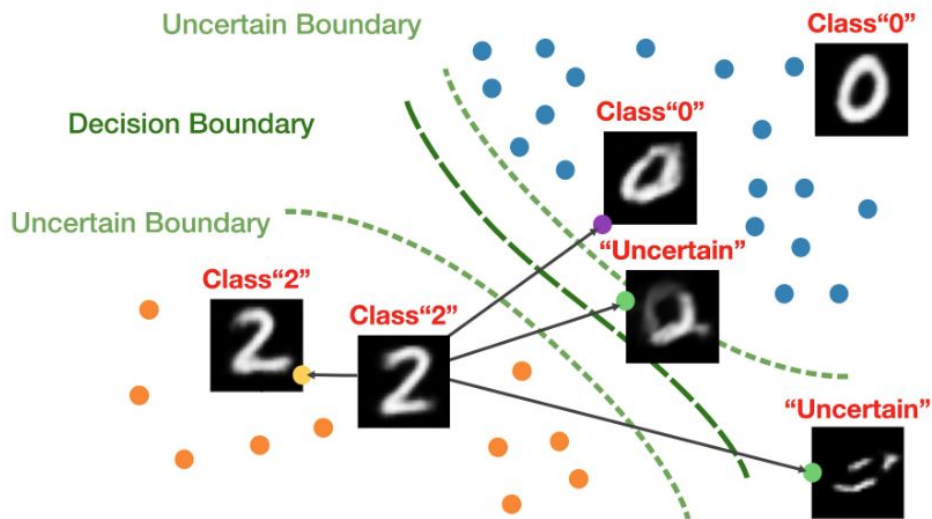


credit: ImageNet-C

## Natural corruptions

# Other dimensions in trustworthy AI

Trustworthiness: robustness, fairness, explainability, transparency



**Boldness: able to change predictions when necessary**

# International concerns and priorities



Administration   Priorities   The Record

OCTOBER 30, 2023

## FACT SHEET: President Biden Issues Executive Order on Safe, Secure, and Trustworthy Artificial Intelligence

 BRIEFING ROOM   STATEMENTS AND RELEASES

Today, President Biden is issuing a landmark Executive Order to ensure that America leads the way in seizing the promise and managing the risks of artificial intelligence (AI). The Executive Order establishes new standards for AI safety and security, protects Americans' privacy, advances equity and civil rights, stands up for consumers and workers, promotes innovation and competition, advances American leadership around the world, and more.

- New Standards for AI Safety and Security
- Protecting Americans' Privacy
- Advancing Equity and Civil Rights
- Standing Up for Consumers, Patients, and Students
- Supporting Workers
- Promoting Innovation and Competition
- Advancing American Leadership Abroad
- Ensuring Responsible and Effective Government Use of AI

<https://www.whitehouse.gov/briefing-room/statements-releases/2023/10/30/fact-sheet-president-biden-issues-executive-order-on-safe-secure-and-trustworthy-artificial-intelligence/>



# Outline

- **Evaluation of adversarial robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Fundamental challenges in evaluating & achieving robustness**

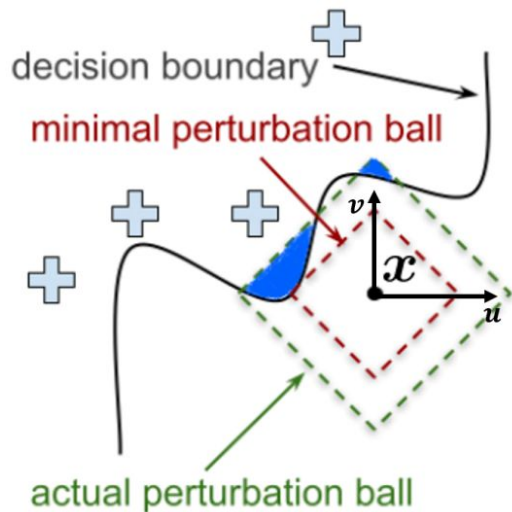
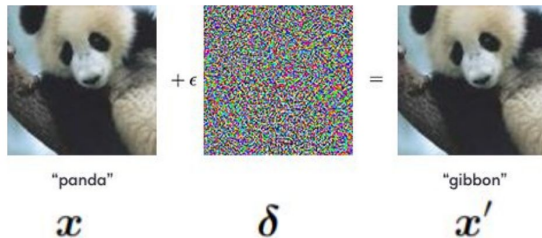
Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Selective prediction**

Selective Classification Under Distribution Shifts (Forthcoming)

- **Closing**

# Empirical robustness evaluation (RE)



Maximize loss/error function

$$\max_{x'} \ell(y, f_{\theta}(x'))$$

$$\text{s. t. } d(x, x') \leq \varepsilon, \quad x' \in [0, 1]^n$$

Allowable perturbation

Valid image

Find robustness radius

$$\min_{x'} d(x, x')$$

$$\text{s. t. } \max_{i \neq y} f_{\theta}^i(x') \geq f_{\theta}^y(x'), \quad x' \in [0, 1]^n$$

On the decision boundary

Valid image

Report robust accuracy over an evaluation set

# Constrained optimization problems

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\boldsymbol{\theta}}(\mathbf{x}')) \\ \text{s. t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

$$\begin{aligned} & \min_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}') \\ \text{s. t. } & \max_{i \neq y} f_{\boldsymbol{\theta}}^i(\mathbf{x}') \geq f_{\boldsymbol{\theta}}^y(\mathbf{x}'), \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

Both objective and constraint functions are **nonconvex** in general, e.g., when containing DL models

# Projected gradient descent (PGD) for RE

$$\max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}'))$$

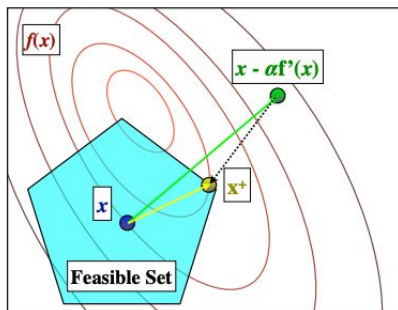
$$\text{s. t. } d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n$$

$$\min_{\mathbf{x} \in \mathcal{Q}} f(\mathbf{x})$$

$$\mathbf{x}_{k+1} = P_{\mathcal{Q}}\left(\mathbf{x}_k - \alpha_k \nabla f(\mathbf{x}_k)\right)$$

Step size

$$P_{\mathcal{Q}}(\mathbf{x}_0) = \arg \min_{\mathbf{x} \in \mathcal{Q}} \frac{1}{2} \|\mathbf{x} - \mathbf{x}_0\|_2^2 \quad \text{Projection operator}$$



Key hyperparameters:

- (1) step size
- (2) iteration number

---

## Algorithm 1 APGD

---

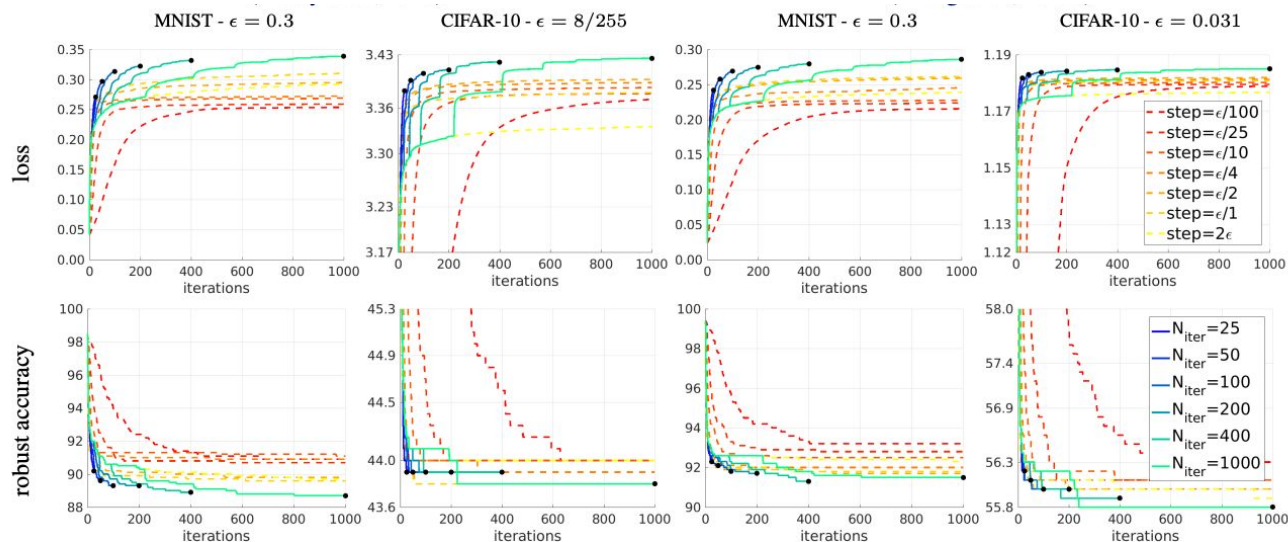
```

1: Input:  $f, S, x^{(0)}, \eta, N_{\text{iter}}, W = \{w_0, \dots, w_n\}$ 
2: Output:  $x_{\max}, f_{\max}$ 
3:  $x^{(1)} \leftarrow P_S(x^{(0)} + \eta \nabla f(x^{(0)}))$ 
4:  $f_{\max} \leftarrow \max\{f(x^{(0)}), f(x^{(1)})\}$ 
5:  $x_{\max} \leftarrow x^{(0)}$  if  $f_{\max} \equiv f(x^{(0)})$  else  $x_{\max} \leftarrow x^{(1)}$ 
6: for  $k = 1$  to  $N_{\text{iter}} - 1$  do
7:    $z^{(k+1)} \leftarrow P_S(x^{(k)} + \eta \nabla f(x^{(k)}))$ 
8:    $x^{(k+1)} \leftarrow P_S(x^{(k)} + \alpha(z^{(k+1)} - x^{(k)}))$ 
      $\quad \quad \quad + (1 - \alpha)(x^{(k)} - x^{(k-1)})$ 
9:   if  $f(x^{(k+1)}) > f_{\max}$  then
10:      $x_{\max} \leftarrow x^{(k+1)}$  and  $f_{\max} \leftarrow f(x^{(k+1)})$ 
11:   end if
12:   if  $k \in W$  then
13:     if Condition 1 or Condition 2 then
14:        $\eta \leftarrow \eta/2$  and  $x^{(k+1)} \leftarrow x_{\max}$ 
15:     end if
16:   end if
17: end for

```

---

# Problem with projected gradient descent



$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ & \text{s.t. } d(\mathbf{x}, \mathbf{x}') \leq \epsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

Tricky to set:  
iteration number & step size  
i.e., **tricky to decide where to stop**



# Penalty methods for complicated d

$$\max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}'))$$

$$\text{s. t. } d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n$$

$$d(\mathbf{x}, \mathbf{x}') \doteq \|\phi(\mathbf{x}) - \phi(\mathbf{x}')\|_2$$

**perceptual  
distance**

$$\text{where } \phi(\mathbf{x}) \doteq [\hat{g}_1(\mathbf{x}), \dots, \hat{g}_L(\mathbf{x})]$$

**Projection onto the constraint is complicated**

## Penalty methods

$$\max_{\tilde{\mathbf{x}}} \mathcal{L}(f(\tilde{\mathbf{x}}), y) - \lambda \max\left(0, \|\phi(\tilde{\mathbf{x}}) - \phi(\mathbf{x})\|_2 - \epsilon\right)$$

Solve it for each fixed  $\lambda$  and then increase  $\lambda$

---

### Algorithm 2 Lagrangian Perceptual Attack (LPA)

---

```

1: procedure LPA(classifier network  $f(\cdot)$ , LPIPS distance  $d(\cdot, \cdot)$ , input  $\mathbf{x}$ , label  $y$ , bound  $\epsilon$ )
2:    $\lambda \leftarrow 0.01$ 
3:    $\tilde{\mathbf{x}} \leftarrow \mathbf{x} + 0.01 * \mathcal{N}(0, 1)$   $\triangleright$  initialize perturbations with random Gaussian noise
4:   for  $i$  in  $1, \dots, S$  do  $\triangleright$  we use  $S = 5$  iterations to search for the best value of  $\lambda$ 
5:     for  $t$  in  $1, \dots, T$  do  $\triangleright T$  is the number of steps
6:        $\Delta \leftarrow \nabla_{\tilde{\mathbf{x}}} [\mathcal{L}(f(\tilde{\mathbf{x}}), y) - \lambda \max(0, d(\tilde{\mathbf{x}}, \mathbf{x}) - \epsilon)]$   $\triangleright$  take the gradient of (5)
7:        $\hat{\Delta} = \Delta / \|\Delta\|_2$   $\triangleright$  normalize the gradient
8:        $\eta = \epsilon * (0.1)^{t/T}$   $\triangleright$  the step size  $\eta$  decays exponentially
9:        $m \leftarrow d(\tilde{\mathbf{x}}, \tilde{\mathbf{x}} + h\hat{\Delta})/h$   $\triangleright m \approx$  derivative of  $d(\tilde{\mathbf{x}}, \cdot)$  in the direction of  $\hat{\Delta}$ ;  $h = 0.1$ 
10:       $\tilde{\mathbf{x}} \leftarrow \tilde{\mathbf{x}} + (\eta/m)\hat{\Delta}$   $\triangleright$  take a step of size  $\eta$  in LPIPS distance
11:    end for
12:    if  $d(\tilde{\mathbf{x}}, \mathbf{x}) > \epsilon$  then
13:       $\lambda \leftarrow 10\lambda$   $\triangleright$  increase  $\lambda$  if the attack goes outside the bound
14:    end if
15:  end for
16:   $\tilde{\mathbf{x}} \leftarrow \text{PROJECT}(d, \tilde{\mathbf{x}}, \mathbf{x}, \epsilon)$ 
17:  return  $\tilde{\mathbf{x}}$ 
18: end procedure

```

---

# Problem with penalty methods

Method	cross-entropy loss		margin loss	
	Viol. (%) ↓	Att. Succ. (%) ↑	Viol. (%) ↓	Att. Succ. (%) ↑
Fast-LPA	73.8	3.54	41.6	56.8
LPA	<b>0.00</b>	80.5	<b>0.00</b>	97.0
PPGD	5.44	25.5	<b>0.00</b>	38.5
PWCF (ours)	0.62	<b>93.6</b>	<b>0.00</b>	<b>100</b>

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s. t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \\ & d(\mathbf{x}, \mathbf{x}') \doteq \|\phi(\mathbf{x}) - \phi(\mathbf{x}')\|_2 \\ \text{where } & \phi(\mathbf{x}) \doteq [\hat{g}_1(\mathbf{x}), \dots, \hat{g}_L(\mathbf{x})] \end{aligned}$$

**LPA, Fast-LPA:** penalty methods

**PPGD:** Projected gradient descent

**PWCF**, an optimizer with a principled stopping criterion on **stationarity** & **feasibility**

Penalty methods tend to encounter

**large constraint violation** (i.e., infeasible solution, known in optimization theory) or **suboptimal solution**

Unreliable optimization = Unreliable RE

# Issues and answers

## projected gradient descent

$$\min_{\mathbf{x} \in \mathcal{Q}} f(\mathbf{x})$$

$$\mathbf{x}_{k+1} = P_{\mathcal{Q}}(\mathbf{x}_k - \alpha_k \nabla f(\mathbf{x}_k))$$

Issue: no principled stopping criterion  
/step size rules

## penalty methods

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{s. t. } g(\mathbf{x}) \leq 0$$

$$\min_{\mathbf{x}} f(\mathbf{x}) + \lambda \max(0, g(\mathbf{x}))$$

Solved with increasing  $\lambda$ : sequence

Issue: infeasible or suboptimal solution

- Feasible & stationary solution **Stationarity and feasibility check: KKT condition**
- Reasonable speed **Line search & 2nd order methods**
- A hidden problem: nonsmoothness

# A principled solver for constrained, nonconvex, nonsmooth problems



**Nonconvex, nonsmooth, constrained**

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \quad \text{s.t. } c_i(\mathbf{x}) \leq 0, \forall i \in \mathcal{I}; \quad c_i(\mathbf{x}) = 0, \forall i \in \mathcal{E}.$$

**Penalty sequential quadratic programming  
(P-SQP)**

$$\begin{aligned} \min_{d \in \mathbb{R}^n, s \in \mathbb{R}^p} \quad & \mu(f(x_k) + \nabla f(x_k)^\top d) + e^\top s + \frac{1}{2} d^\top H_k d \\ \text{s.t.} \quad & c(x_k) + \nabla c(x_k)^\top d \leq s, \quad s \geq 0, \end{aligned}$$

**Quasi-Newton methods (L-BFGS) → high-precision solution**

**Principled line search, stationarity/feasibility check**

**Ref** Curtis, Frank E., Tim Mitchell, and Michael L. Overton. "A BFGS-SQP method for nonsmooth, nonconvex, constrained optimization and its evaluation using relative minimization profiles." *Optimization Methods and Software* 32.1 (2017): 148-181.



# Our PyGranso (and NCVX framework)

GRANVISO + PyTorch



NCVX PyGRANSO  
Documentation

Search the docs ...

Introduction

Installation

Settings

Examples

Home



NCVX Package

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \text{ s.t. } c_i(\mathbf{x}) \leq 0, \forall i \in \mathcal{I}; c_i(\mathbf{x}) = 0, \forall i \in \mathcal{E}$$

**First general-purpose solver for  
constrained DL problems**

**NCVX: A General-Purpose Optimization Solver for  
Constrained Machine and Deep Learning**

Buyun Liang, Tim Mitchell, Ju Sun

<https://ncvx.org/>

# Strategies to speed up PyGranso for RE

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s. t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

**Constraint folding: many constraints into few**

$$\begin{aligned} h_j(\mathbf{x}) = 0 & \iff |h_j(\mathbf{x})| \leq 0, \\ c_i(\mathbf{x}) \leq 0 & \iff \max\{c_i(\mathbf{x}), 0\} \leq 0, \\ \mathcal{F}(|h_1(\mathbf{x})|, \dots, |h_i(\mathbf{x})|, \max\{c_1(\mathbf{x}), 0\}, \\ & \dots, \max\{c_j(\mathbf{x}), 0\}) \leq 0, \end{aligned}$$

$$\begin{aligned} & \min_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}') \\ \text{s. t. } & \max_{i \neq y} f_{\theta}^i(\mathbf{x}') \geq f_{\theta}^y(\mathbf{x}'), \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

**Two-stage optimization**


1. **Stage 1 (selecting the best initialization):** Optimize the problems by PWCF with  $R$  different random initialization  $\mathbf{x}^{(r,0)}$  for  $k$  iterations, where  $r = 1, \dots, R$ , and collect the final first-stage solution  $\mathbf{x}^{(r,k)}$  for each run. Determine the best intermediate result  $\mathbf{x}^{(*,k)}$  following [Algorithm 1](#).
2. **Stage 2 (optimization):** Warm start the optimization process with  $\mathbf{x}^{*,k}$  until the stopping criterion is met [7](#) (i.e., reaching both the stationarity and feasibility tolerance, or reaching the MaxIter  $K$ ).

# First general-purpose, reliable solver for RE

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s. t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

$$\begin{aligned} & \min_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}') \\ \text{s. t. } & \max_{i \neq y} f_{\theta}^i(\mathbf{x}') \geq f_{\theta}^y(\mathbf{x}'), \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

## Reliability

- SOTA methods   
No stopping criterion (only use maxit); step size scheduler
- PWCF (ours)  
Principled line-search rule and termination criterion

## Generality

- SOTA methods  
Can mostly only handle several lp metrics (l1, l2, linf)
- PWCF (ours)  
Any almost everywhere differentiable metrics and both min and max forms  
E.g., perceptual distance  
$$d(\mathbf{x}, \mathbf{x}') \doteq \|\phi(\mathbf{x}) - \phi(\mathbf{x}')\|_2$$
  
where  $\phi(\mathbf{x}) \doteq [\hat{g}_1(\mathbf{x}), \dots, \hat{g}_L(\mathbf{x})]$

# A quick example

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s.t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned} \quad \text{where} \quad \begin{aligned} & d(\mathbf{x}, \mathbf{x}') \doteq \|\phi(\mathbf{x}) - \phi(\mathbf{x}')\|_2 \\ & \phi(\mathbf{x}) \doteq [\hat{g}_1(\mathbf{x}), \dots, \hat{g}_L(\mathbf{x})] \end{aligned}$$

Method	cross-entropy loss		margin loss	
	Viol. (%) ↓	Att. Succ. (%) ↑	Viol. (%) ↓	Att. Succ. (%) ↑
Fast-LPA	73.8	3.54	41.6	56.8
LPA	<b>0.00</b>	80.5	<b>0.00</b>	97.0
PPGD	5.44	25.5	<b>0.00</b>	38.5
PWCF (ours)	0.62	<b>93.6</b>	<b>0.00</b>	<b>100</b>

# PyGranso has enabled much more

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \text{ s.t. } c_i(\mathbf{x}) \leq 0, \forall i \in \mathcal{I}; c_i(\mathbf{x}) = 0, \forall i \in \mathcal{E}$$

**First general-purpose solver for constrained DL problems**

 **PyGRANSO**  
NCVX PyGRANSO  
Documentation

🔍 Search the docs ...

Introduction

Installation

Settings

Examples



Home



NCVX Package

**NCVX: A General-Purpose Optimization Solver for Constrained Machine and Deep Learning**

Buyun Liang, Tim Mitchell, Ju Sun

**Topology optimization**

$$\min_{\theta, \mathbf{u}} \mathbf{u}^\top \mathbf{K}(\mathbf{G}_\theta(\beta)) \mathbf{u} \quad \text{s.t.} \quad \mathbf{K}(\mathbf{G}_\theta(\beta)) \mathbf{u} = \mathbf{f},$$

$$\sum_{i \in \Omega} [\mathbf{G}_\theta(\beta)]_i = v_0, \quad \mathbf{G}_\theta(\beta) \in \{0, 1\}^n$$

**Imbalanced learning**

$$\max_{\theta, t} \frac{\sum_{i=1}^N \mathbb{1}\{y_i = +1\} \mathbb{1}\{f_\theta(\mathbf{x}_i) > t\}}{\sum_{i=1}^N \mathbb{1}\{f_\theta(\mathbf{x}_i) > t\}}$$

$$\text{s.t.} \quad \frac{\sum_{i=1}^N \mathbb{1}\{y_i = +1\} \mathbb{1}\{f_\theta(\mathbf{x}_i) > t\}}{\sum_{i=1}^N \mathbb{1}\{y_i = +1\}} \geq \alpha$$

**Constrained deep learning for the efficient discovery of stable solid-state materials**

PIs: Chris Bartel (CEMS), Ju Sun (CS&E)

**Background**

Machine/deep learning (MDL) has emerged as a novel tool in material science and engineering (MSE).<sup>1</sup> MDL models in MSE can be broadly categorized as “property prediction models” (PPMs) or “interatomic potentials” (IPs). For the former, the goal is to learn the mapping between material representations and material properties (e.g., formation energy, band gap, etc.). These representations can be compositional,<sup>2</sup> requiring only the chemical formula (e.g.,  $\text{Al}_2\text{O}_3$ ), or structural,<sup>3</sup> requiring the formula and the 3D arrangement of ions on a periodic lattice (e.g.,  $\text{Al}_2\text{O}_3$  in the corundum structure with specified coordinates for Al and O). IPs make use of a structural representation, but instead of learning to predict a single property, these models learn to predict the energies, forces, and stresses of an arbitrary configuration of ions on a lattice.<sup>4</sup> Using this learned interatomic model, one can perform a set of tasks and analyses that are usually



# Outline

- **Evaluation of adversarial robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Fundamental challenges in evaluating & achieving robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

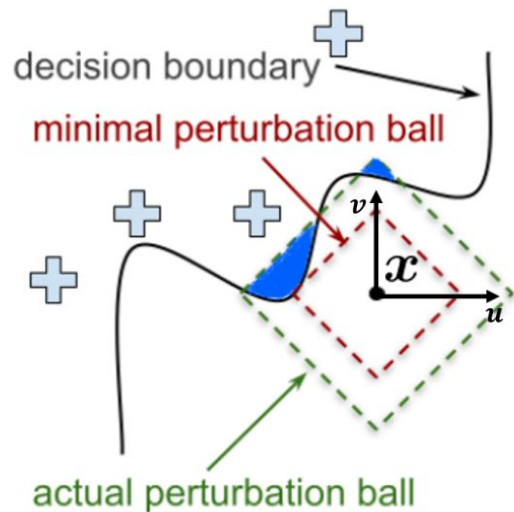
- **Selective prediction**

Selective Classification Under Distribution Shifts (Forthcoming)

- **Closing**

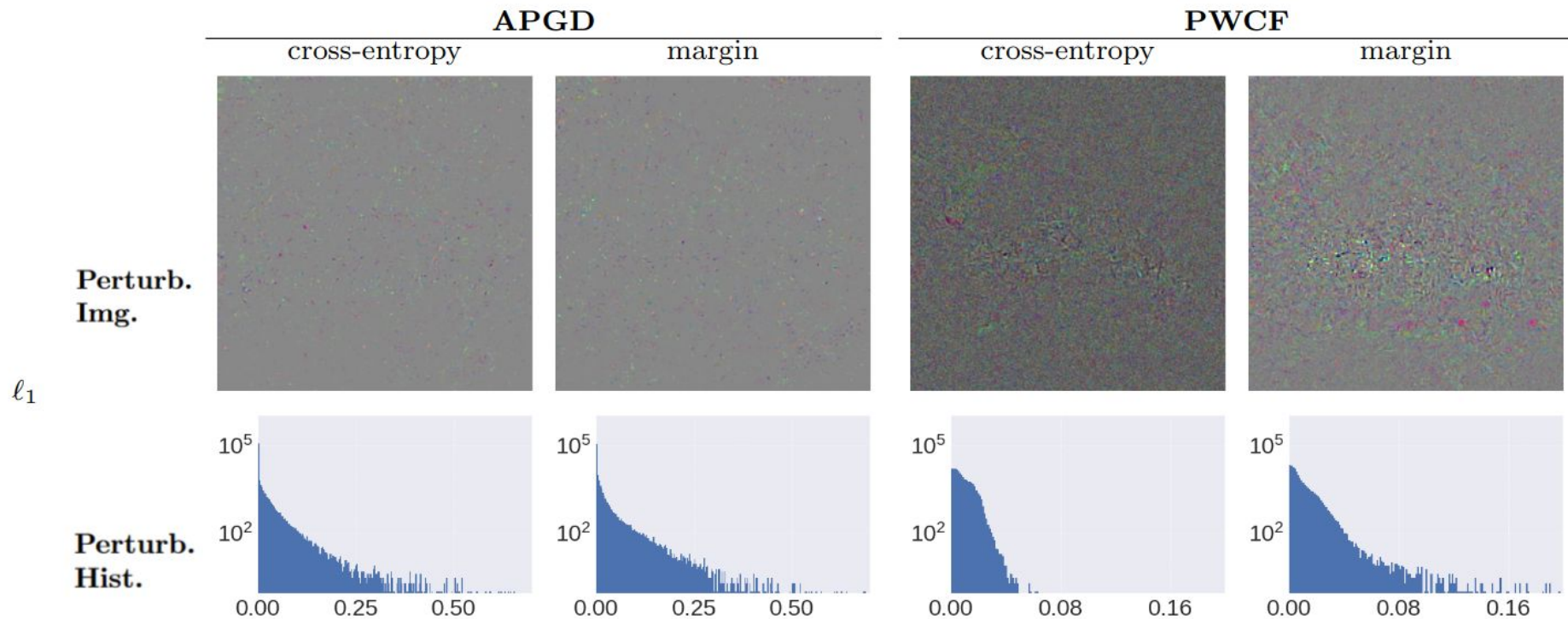
# RE tractable even with PW/CF?

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s.t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$



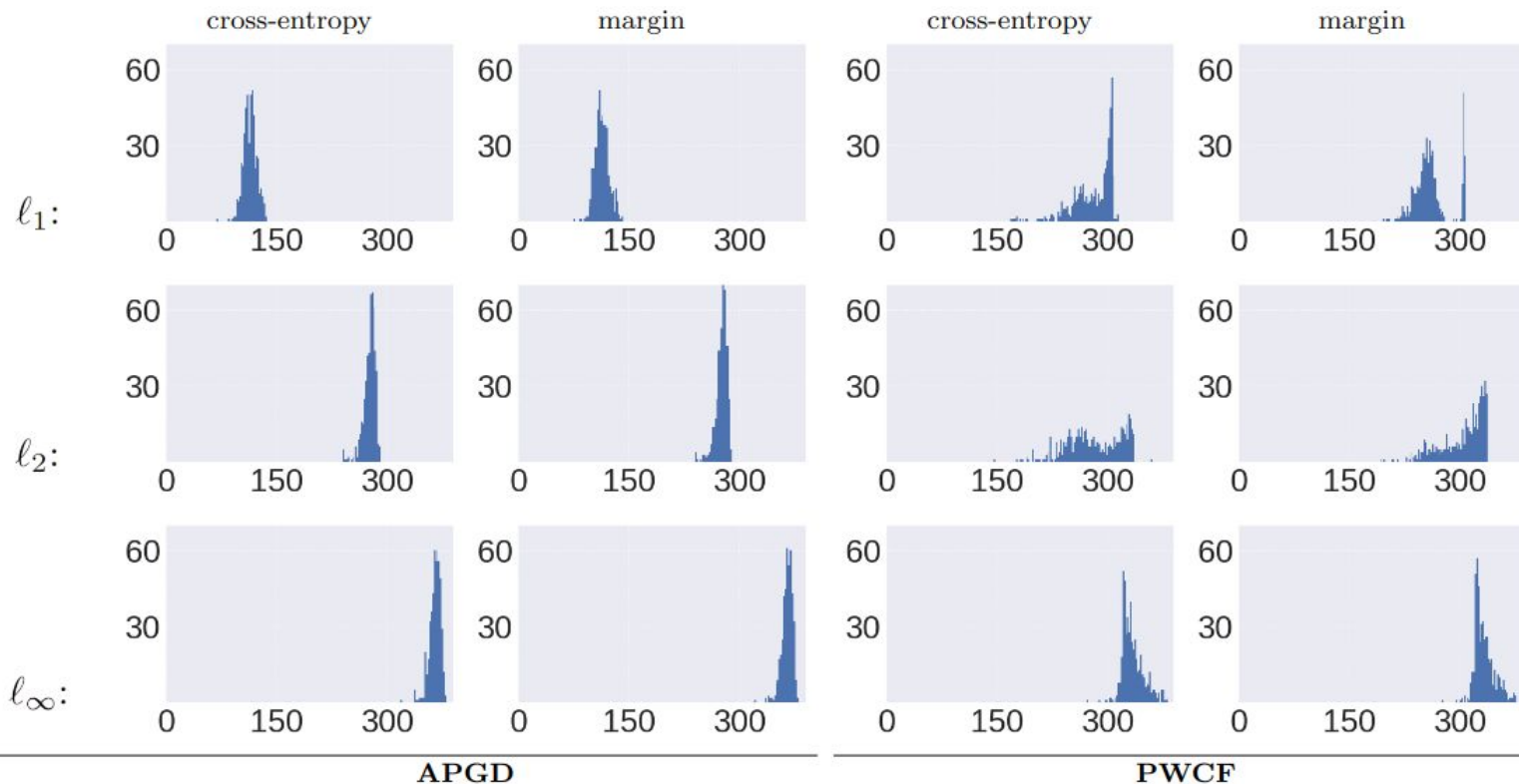
- Assuming 0-1 loss
- Typical **over-specification of  $\varepsilon$**  means there are potentially infinitely many solutions, with **different patterns**

# Is the intuition right?



# Is the intuition right?

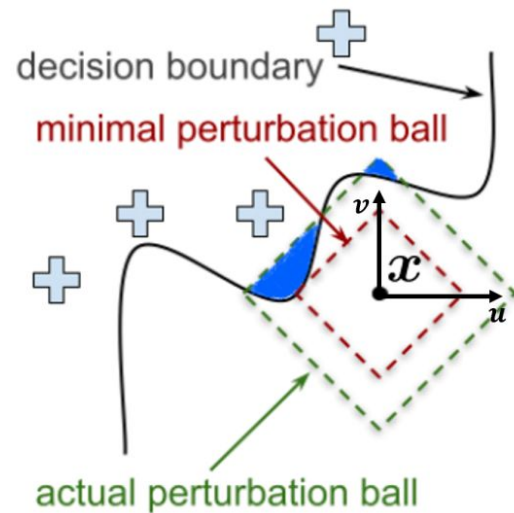
Measured by **sparsity levels** of the perturbations found



## Implications - I

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s. t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

We need to **enumerate** all possible solutions if we want reliable RE using max-form



**Take-away: Max-form RE is fundamentally intractable, unless a good  $\varepsilon$  is set—which is hard**

## Implications - II

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s. t. } & \boxed{d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n} \quad \mathbf{x}' \in \Delta(\mathbf{x}) \end{aligned}$$

**Adversarial training**  $\min_{\theta} \mathbb{E}_{(\mathbf{x}, \mathbf{y}) \sim \mathcal{D}} \max_{\mathbf{x}' \in \Delta(\mathbf{x})} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}'))$

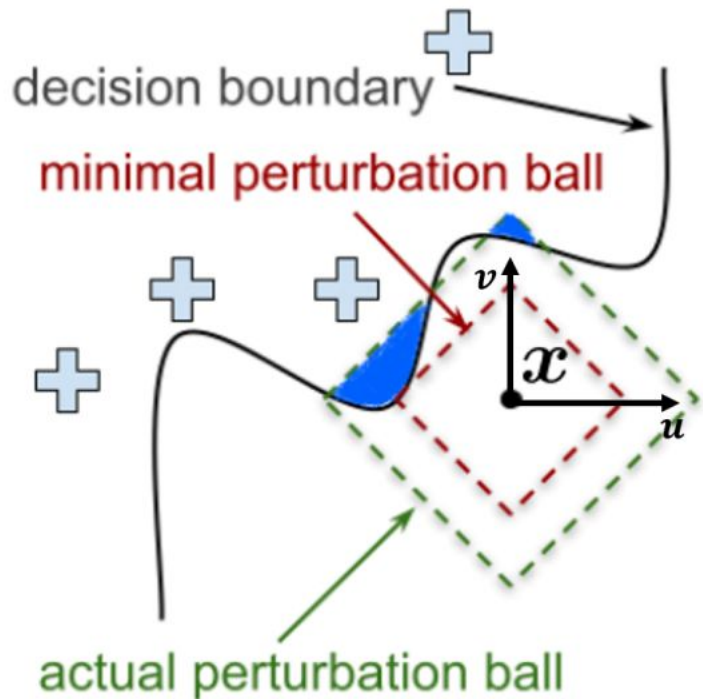
i.e., data augmentation with adversarial samples

We need to **enumerate** all possible patterns of adversarial samples if we want to achieve robustness, measured by the same  $d$

**Take-away: Adversarial training with the max-form augmentation won't achieve robustness**



# Any hopes remaining?



$$\begin{aligned} & \max_{x'} \ell(y, f_{\theta}(x')) \\ \text{s. t. } & d(x, x') \leq \varepsilon, \quad x' \in [0, 1]^n \end{aligned}$$

VS

$$\begin{aligned} & \min_{x'} d(x, x') \\ \text{s. t. } & \max_{i \neq y} f_{\theta}^i(x') \geq f_{\theta}^y(x'), \quad x' \in [0, 1]^n \end{aligned}$$

**Take-away: the min-form  
(robustness radius) is  
more promising**

# Outline

- **Evaluation of adversarial robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Fundamental challenges in evaluating & achieving robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Selective prediction**

Selective Classification Under Distribution Shifts (Forthcoming)

- **Closing**

# We have a long way to go



## TRUSTWORTHY AI RESEARCH THRUSTS

DARPA experts estimate that research in the following areas will be essential to creating trustworthy technology:

- Foundational theory, to understand the art of the possible, bound the limits of particular system instantiations, and inform guardrails for AI systems in challenging domains such as national security;
- AI engineering, to predictably build systems that work as intended in the real world and not just in the lab; and
- Human-AI teaming, to enable systems to serve as fluent, intuitive, trustworthy teammates to people with various backgrounds.

<https://www.darpa.mil/work-with-us/ai-forward>

## Safe Learning-Enabled Systems

### PROGRAM SOLICITATION NSF 23-562



#### National Science Foundation

Directorate for Computer and Information Science and Engineering  
Division of Information and Intelligent Systems  
Division of Computing and Communication Foundations  
Division of Computer and Network Systems



Open Philanthropy Project LLC



Good Ventures Foundation

**Full Proposal Deadline(s)** (due by 5 p.m. submitter's local time):

May 26, 2023

January 16, 2024

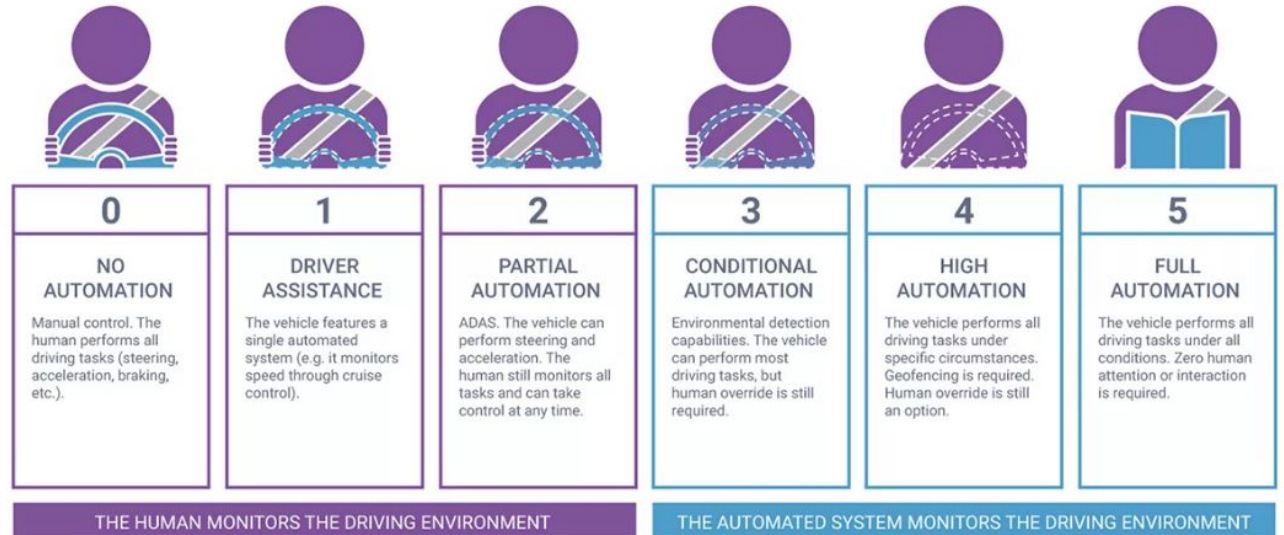
<https://www.nsf.gov/pubs/2023/nsf23562/nsf23562.htm>

# Imperfect AI models can still be deployed



SYNOPSIS®

## LEVELS OF DRIVING AUTOMATION



A crucial component: allowing AI to restrain itself

predictor  $f : \mathcal{X} \rightarrow \mathbb{R}^K$     selector  $g : \mathcal{X} \rightarrow \{0, 1\}$

$$(f, g)(\mathbf{x}) \triangleq \begin{cases} f(\mathbf{x}) & \text{if } g(\mathbf{x}) = 1; \\ \text{abstain} & \text{if } g(\mathbf{x}) = 0. \end{cases}$$

No prediction on uncertain samples and defer them to humans

$$g_{\gamma}(\mathbf{x}) = \mathbb{1}[s(\mathbf{x}) > \gamma]$$

Typically, selection by thresholding prediction confidence

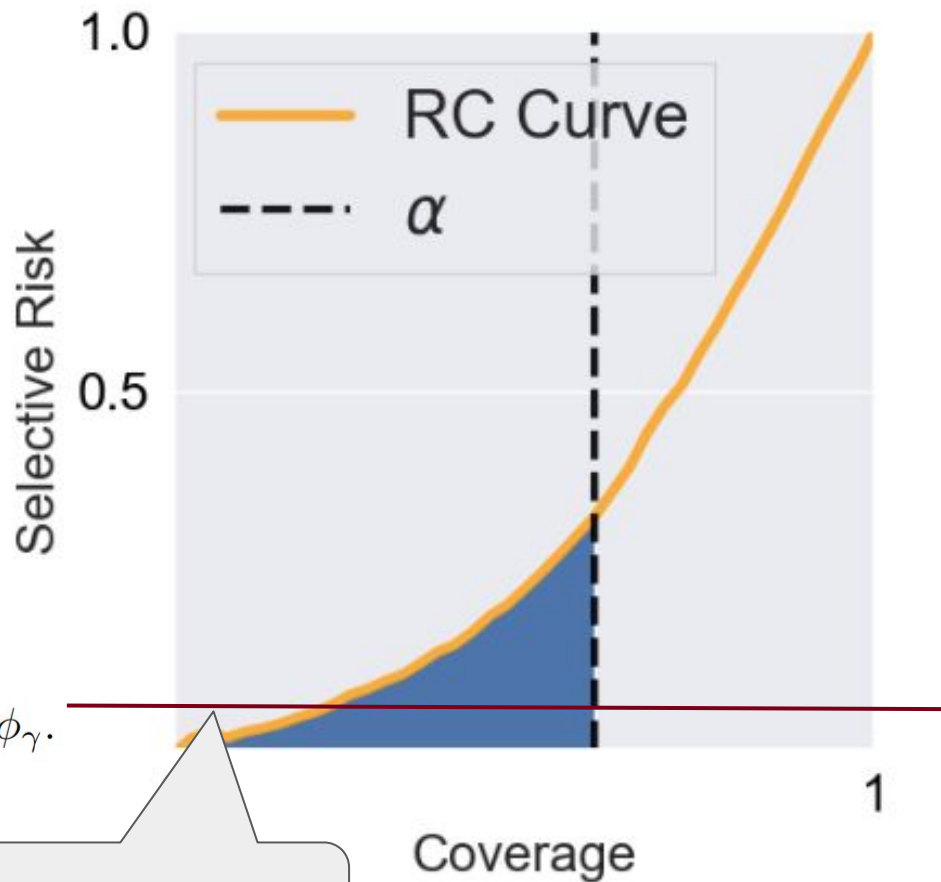
# Risk-coverage tradeoff

$$(f, g)(\mathbf{x}) \triangleq \begin{cases} f(\mathbf{x}) & \text{if } g(\mathbf{x}) = 1; \\ \text{abstain} & \text{if } g(\mathbf{x}) = 0. \end{cases}$$

$$g_\gamma(\mathbf{x}) = \mathbb{1}[s(\mathbf{x}) > \gamma]$$

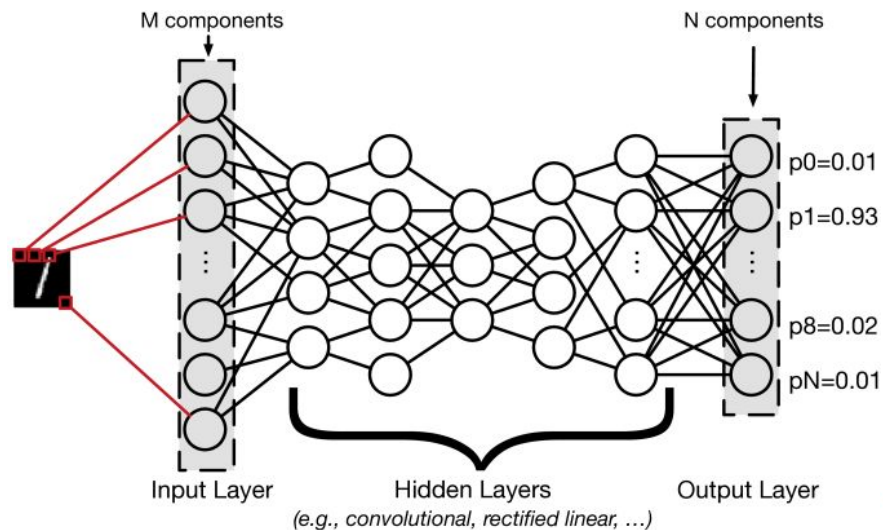
(coverage)  $\phi_\gamma = \mathbb{E}_{\mathcal{D}}[g_\gamma(\mathbf{x})]$ ,

(selection risk)  $R_\gamma = \mathbb{E}_{\mathcal{D}}[\ell(f(\mathbf{x}), y)g_\gamma(\mathbf{x})]/\phi_\gamma$ .



**High-stakes corner**

# Which confidence score?



$$\approx p(y = 1|x)$$

$\mathbf{z} \in \mathbb{R}^K$  contains the raw logits (RLs)

$$SR_{\max} \triangleq \max_i \sigma(z^i),$$

$$SR_{\text{doctor}} \triangleq (\|\sigma(\mathbf{z})\|_2^2 - 1) / \|\sigma(\mathbf{z})\|_2^2 = 1 - \|\sigma(\mathbf{z})\|_1 / \|\sigma(\mathbf{z})\|_2^2,$$

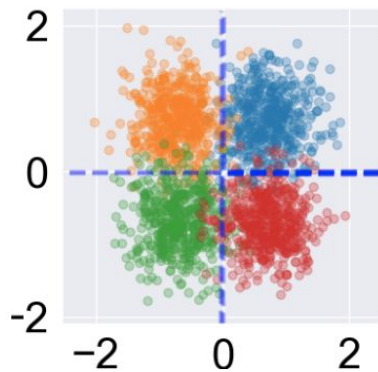
$$SR_{\text{ent}} \triangleq \sum_i \sigma(z^i) \log \sigma(z^i),$$



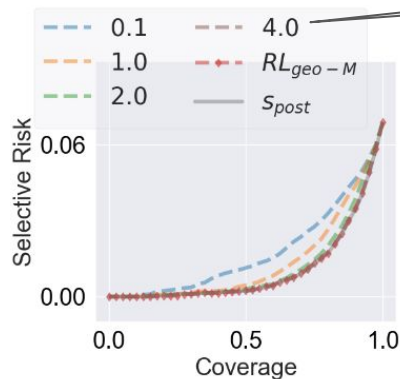
# But are they good scores?

$z \in \mathbb{R}^K$  contains the raw logits (RLs)

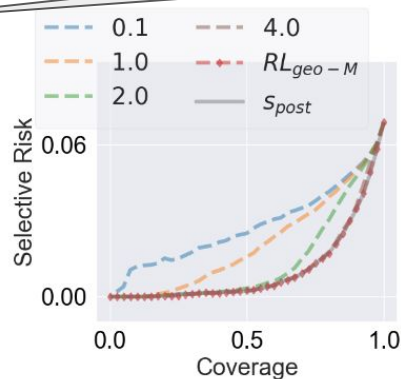
Scale factor applied to RLs



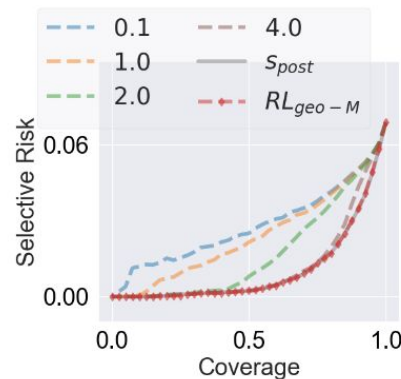
(a) Sample visualization



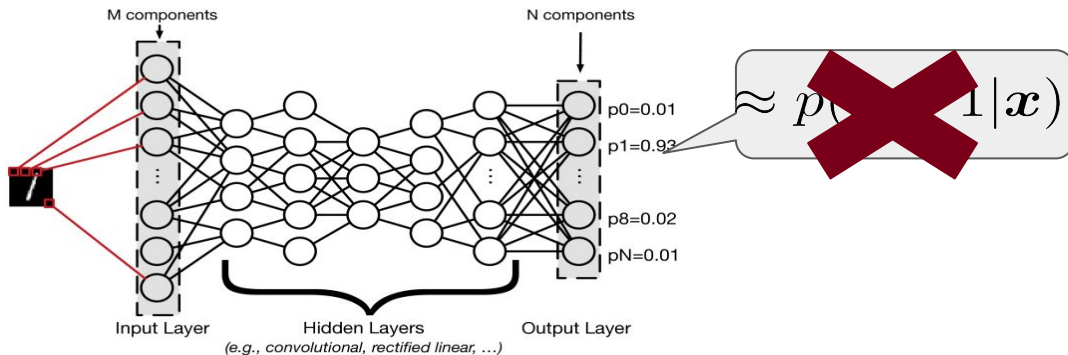
(b)  $SR_{\max}$



(c)  $SR_{\text{doctor}}$



(d)  $SR_{\text{ent}}$



**Calibration: align the outputs with the true posterior probs**

# Our margin-based scores

Signed dist to the separating hyperplane

Binary SVMs:  $f(x) = w^\top x + b$

Geometric margin:  $y(w^\top x + b) / \|w\|_2$

Multiclass SVMs:  $f(x) = W^\top x + b$

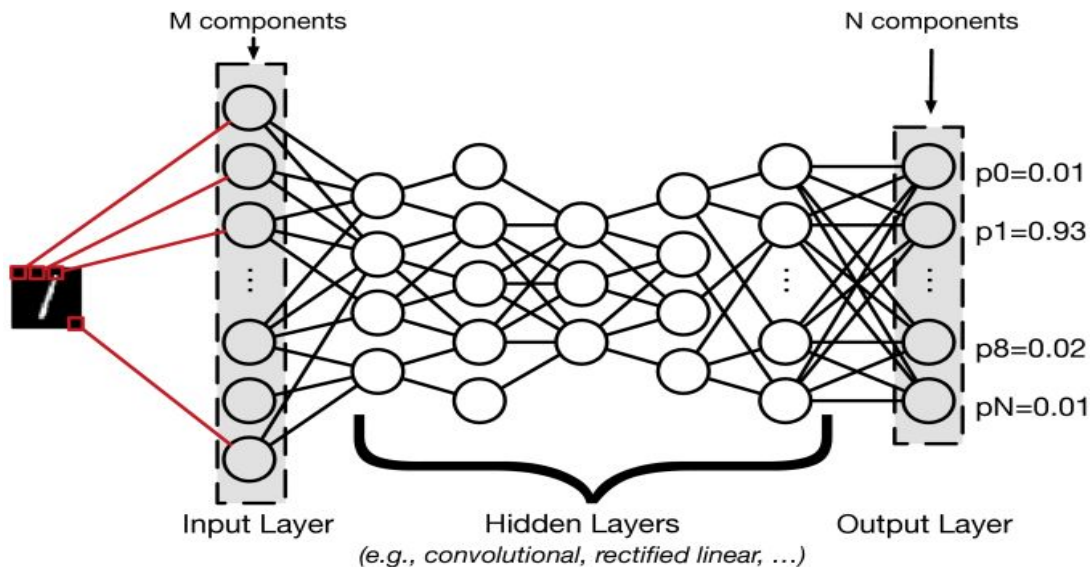
Geometric margin:  $\frac{w_{y'}^\top x + b_{y'}}{\|w_{y'}\|_2} - \max_{j \in \{1, \dots, K\} \setminus y'} \frac{w_j^\top x + b_j}{\|w_j\|_2}$

Confidence margin:  $(w_{y'}^\top x + b_{y'}) - \max_{i \in \{1, \dots, K\} \setminus y'} (w_i^\top x + b_i)$

Difference of dists between the two nearest hyperplanes

**These scores are not affected by the logit scaling**

# Our margin-based scores



**Geometric margin:**

$$\frac{\mathbf{w}_{y'}^T \mathbf{x} + b_{y'}}{\|\mathbf{w}_{y'}\|_2} - \max_{j \in \{1, \dots, K\} \setminus y'} \frac{\mathbf{w}_j^T \mathbf{x} + b_j}{\|\mathbf{w}_j\|_2}$$

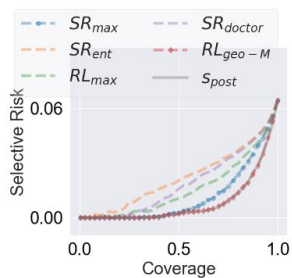
**Confidence margin:**

$$(\mathbf{w}_{y'}^T \mathbf{x} + b_{y'}) - \max_{i \in \{1, \dots, K\} \setminus y'} (\mathbf{w}_i^T \mathbf{x} + b_i)$$

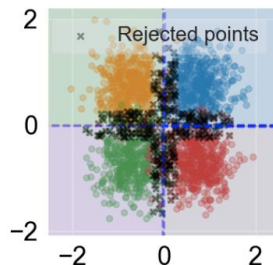
**Apply them to the RLs  $\mathcal{Z}$**

Benefit: We don't need to worry about the scale of  $\mathcal{Z}$

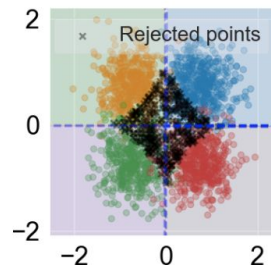
# Additional benefit: robustness



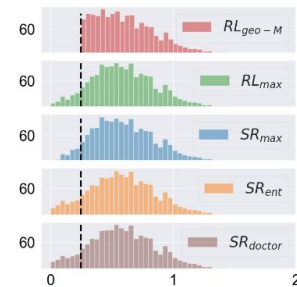
(a-1) RC curves



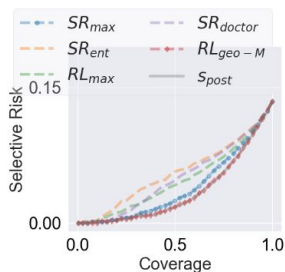
(b-1)  $RL_{geo-M}$



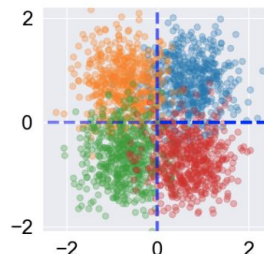
(c-1)  $SR_{max}$



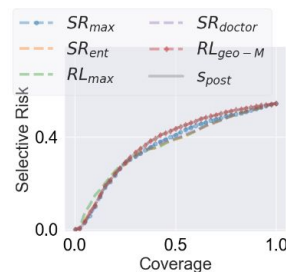
(d-1) Robustness radius



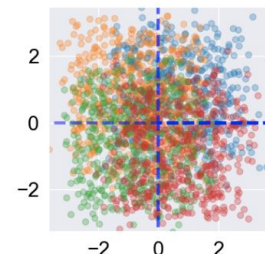
(a-2) RC curves



(b-2) Sample visualization



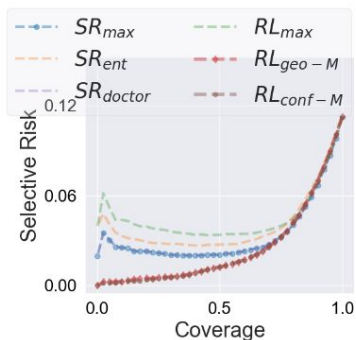
(a-3) RC curves



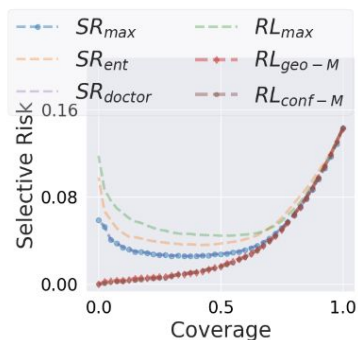
(b-3) Sample visualization

# On real data

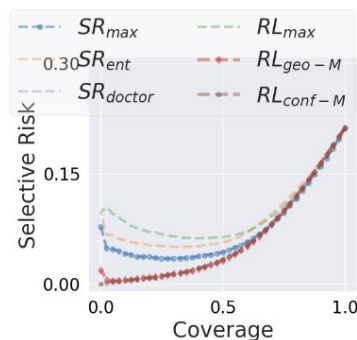
ImageNet vs ImageNet-C



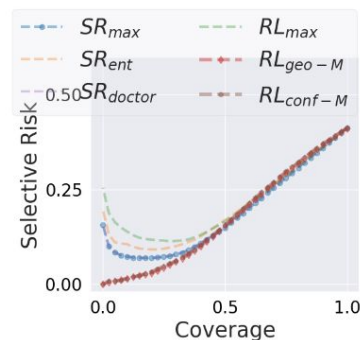
(a) IN (Clean)



(b) Gaussian blur Lv.1



(c) Gaussian blur Lv.3



(d) Gaussian blur Lv.5

	IN (Clean)			Gaussian Blur			Brightness			Fog			Snow		
$\alpha$	0.1	0.5	1	0.1	0.5	1	0.1	0.5	1	0.1	0.5	1	0.1	0.5	1
$RL_{\text{conf-M}}$	<b>0.16</b>	<b>0.53</b>	<b>2.39</b>	<b>0.37</b>	<b>1.31</b>	<u>6.05</u>	<b>0.21</b>	<b>0.72</b>	<b>3.35</b>	<b>0.14</b>	<b>0.79</b>	<b>4.21</b>	<b>0.17</b>	<b>0.95</b>	<b>4.80</b>
$RL_{\text{geo-M}}$	<u>0.27</u>	<u>0.59</u>	<u>2.43</u>	<u>0.57</u>	<u>1.36</u>	<b>6.04</b>	<u>0.33</u>	<u>0.79</u>	<u>3.39</u>	<u>0.25</u>	<u>0.86</u>	<u>4.22</u>	<u>0.34</u>	<u>1.02</u>	<u>4.81</u>
$RL_{\text{max}}$	5.54	4.05	4.57	9.74	7.38	9.52	7.38	5.17	6.06	7.74	5.77	7.01	9.44	6.44	7.90
$SR_{\text{max}}$	3.19	2.40	3.38	5.02	4.02	7.39	4.07	2.90	4.53	3.92	3.07	5.37	5.35	3.67	6.13
$SR_{\text{ent}}$	4.28	3.13	4.04	6.80	5.63	8.71	5.51	4.01	5.48	5.56	4.37	6.42	7.29	5.07	7.27
$SR_{\text{doctor}}$	3.21	2.38	3.40	5.05	4.05	7.47	4.10	2.93	4.58	3.95	3.10	5.42	5.39	3.71	6.20

# Outline

- **Evaluation of adversarial robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Fundamental challenges in evaluating & achieving robustness**

Optimization and Optimizers for Adversarial Robustness <https://arxiv.org/abs/2303.13401>

- **Selective prediction**

Margin As An Effective Confidence Score For Selective Classification Under Distribution Shifts  
(Forthcoming)

- **Closing**



# Closing

- A long way to go for DL robustness
- Selective prediction crucial for deploying imperfect AI

