Question 1: What are the contributions of this work?

Existing Work:
- Uniform Progress
- LP Rounding Succeeds
- Stationary Point

Algorithm based on Alternating Direction Method (ADM):
- Minimize $\min_x Y_q - x_1 + \lambda x_1$, s.t. $|q|_1 = 1$.
- Use alternating direction to find $x_{k+1}$.

Theorem (Exact Recovery for the ADM Algorithm, PSV):
- Apply the ADM algorithm (5) with $\lambda = 1/\sqrt{\theta}$, using all rows of $Y$ as initializations for $q^{(0)}$ to produce $q_1, \ldots, q_n$. Solve the LP rounding (6) with $r = q_1, \ldots, q_n$ to produce $q_1, \ldots, q_n$. Set $r \in \arg\min_n |Y Q q|_2$, with very high probability, $|Y Q q|_2 \geq 2 \sqrt{\theta}$ for some $n \neq 0$, provided $p \geq \sqrt{(n \log n)}$, and $\theta \leq \epsilon$.

A Sketch of Analysis:
- Under the PSV model, let $q = [q_1, q_2, \ldots, q_n]$, $G = [g_1, \ldots, g_{n-1}]$. Assume the orthonormal matrix $Y = [Y_1, Y_2, \ldots, Y_n] = \sum_{i=1}^{n-1} q_i(q_i \cos)_{G}$. Define a random process over $q \in S^{n-1}$.
- Good initialization: One of initializers $q^{(2)} = y^\perp$, w.h.p., $\|q^{(2)} - q^{(1)}\|_2 \leq 3(\sqrt{\theta})$.
- Uniform progress away from the equator: $\|q^{(2)} - q^{(0)}\|_2 \leq \sqrt{3(\sqrt{\theta})} - 3/4 \leq 3/4 \leq \sqrt{\theta}$, w.h.p.
- No jumps away from the cap: For all $q \in S^{n-1}$ with $\frac{\sqrt{n}}{\sqrt{\theta}} \leq \|q\|_2 \leq 3\sqrt{\theta}$, w.h.p., $\|Q q\|_2 > 0$.
- Location of the stationary point: Steps above implies that if the ADM algorithm starts from a point $q^{(0)}$ with $|q^{(0)}|_2 > \sqrt{\theta}$, it will converge to a stationary point $q$ such that $|q|_2 > \sqrt{2\theta}$.
- LP rounding succeeds: Solving (6) with $r = q$, w.h.p., will output a solution $\hat{q}$.

Discussions:
- More Application Ideas
- Intriguing Experiments on Dictionary Learning:
  - Efficient algorithms can also achieve linear sparsity regime for the squared dictionary learning under the Bernoulli-Gaussian model.
  - Generalization: Can we develop general tools for nonconvex function, $\lambda^{i}$ smooth manifold.
  - Nonconvex Problems as a Whole: Phase retrieval, matrix/sensor completion, robust PCA, blind deconvolution, etc.